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Hierarchies of factor solutions in the intelligence domain: Applying methodology from personality psychology to gain insights into the nature of intelligence

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Research on the structure of psychometric intelligence has used hierarchical models like the higher-order and the bi-factor model and has studied the hierarchical relationship between factors within these models. In contrast, research on the structure of personality has not only used hierarchical models but has also studied hierarchies of factor solutions. We clarify the theoretical and conceptual differences between hierarchical models and the solutions-hierarchy approach used in the field of personality research, and suggest that the solutions-hierarchy perspective can provide a novel perspective for intelligence research. We used the solutions-hierarchy approach to study four correlation matrices ($N = 230$ to 710; 38 to 63 tests), and a large dataset ($N = 16{,}823$; 44 tests). Results provided (a) insights into relationships between intelligence constructs across the hierarchy of factor solutions, and (b) evidence that intelligence has a 1–2–3–5 hierarchy of factor solutions with a g factor at the top, gc and gf factors at the second level, a speed–reasoning–knowledge taxonomy at the third level, and possibly a speed-reasoning–fluency–knowledge–memory/perception taxonomy at the fifth level.

Researchers have long been engaged in efforts to find taxonomies for the major domains of human individual differences. Although a consensus on the structure of a domain of individual differences is not necessarily a prerequisite for scientific progress, knowledge on the structure of a domain of individual differences is commonly helpful for integrating findings within a field and for developing a shared scientific language (e.g., Goldberg, 1993; Goldstein, Zedeck, & Goldstein, 2002; John, Naumann, & Soto, 2008).

Research on the structure of psychometric intelligence started in the first half of the last century (e.g., Spearman, 1904, 1927; Thurstone, 1938a, 1938b). In the following decades, researchers developed a variety of different taxonomies (see Carroll, 1993, for an overview). Over time, most researchers reached a consensus on the idea that an optimal taxonomy for the intelligence domain should be a hierarchical structure with one or more broad abilities at the apex of the hierarchy and one or more levels of narrower abilities arranged below the broad abilities (Lubinski, 2004). This progress notwithstanding, research on the structure of psychometric intelligence is still an active field of research and researchers have continued to investigate the characteristics of the factors at each level of the hierarchy (Carroll, 2003; Goldstein et al., 2002; Johnson & Bouchard, 2005; McGrew, 2009).

A notable difference between studies on the structure of intelligence and recent studies on the general structure of personality is that personality researchers frequently use some techniques and conceptualizations of hierarchy that have not yet been employed in intelligence research. Intelligence research typically relies on two types of hierarchical factor models: the higher-order model and the bi-factor (also known as the nested-factors and the hierarchical) model (e.g., Jensen & Weng, 1994; Yung, Thissen, & McLeod, 1999). Personality researchers have also used these hierarchical models but have additionally used a conceptually different approach that focuses on studying hierarchies of factor solutions (Ashton, Lee, & Goldberg, 2004; de Raad & Barelds, 2008; Markon, 2009; Markon et al., 2005; Saucier, 2009; Saucier & Goldberg, 2001; Zuckerman, Kuhlman, & Camac, 1988). This solutions-hierarchy approach entails a different conceptualization of hierarchy and is frequently used by personality researchers when the focus is on understanding and describing the structure of large datasets. The solutions-hierarchy approach is also sometimes referred to as top-down factor analysis in the personality literature (Ashton, Lee, & Goldberg, 2004; Goldberg, 2006; Waller, 2007).

In this article, we seek to build on personality research and suggest that the methodology frequently used in research on the structure of personality—studying hierarchies of factor solutions—also has implications for research on the structure of intelligence. Our goal is to investigate these implications. We begin this article by clarifying the theoretical and conceptual differences between the hierarchical factor

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models commonly used in intelligence research, and the characteristics of the solutions-hierarchy approach used in personality research. We follow up this conceptual section with analyses of five large datasets on intelligence using the solutions-hierarchy approach. Our article contributes to the literature by (a) clarifying different conceptualizations of hierarchy, by (b) establishing a conceptual link between research on the structure of personality and research on the structure of intelligence, and by (c) complementing existing studies and reviews on the structure of intelligence (e.g., Carroll, 1993).

1. Extant conceptualizations of hierarchy in intelligence research

1.1. The higher-order model

One conceptualization of hierarchy that is frequently used in intelligence research is the higher-order model (e.g., Jensen & Weng, 1994; Yung et al., 1999). Fig. 1a shows a simple higher-order model. In this model, a broad second-order general factor influences three narrower abilities which in turn influence the measurement indicators or tests. One important assumption of this model is the idea that the second-order general factor causally influences the narrower abilities. Another characteristic assumption of the model is that the second-order general factor is not directly associated with the manifest tests or measurement indicators at the lowest level. The effect of the second-order general factor on the tests is mediated by the narrower (or first-order) factors. As a result, the second-order factor in the higher-order model has shared variance with the narrower abilities and this shared variance between the two is assigned to the higher-order factor as the causal source of this variance. The second-order factor also does not share variance with the measurement indicator (the test) that is not also shared between narrower abilities and the measurement indicator.

The higher-order model developed from scientific debate between Louis Thurstone (1939; Thurstone & Thurstone, 1941) and Charles Spearman (1939). Spearman had long argued that the general factor extracted from a large intelligence test battery is a sort of mental energy that is responsible for correlations between tests (Spearman, 1904). In his original two-factor model, Spearman used only one latent variable for the general factor and suggested that the variance in each intelligence test consists of variance due to the general factor g and a specific component that is unique to the specific test. Spearman's two-factor theory is similar to a one-factor model (Harman, 1976; Jensen & Weng, 1994). Thurstone, in contrast, developed a multidimensional view of intelligence and preferred to extract oblique (correlated) factors from intelligence data. Spearman (1939) reanalyzed one of Thurstone's datasets using his two-factor theory. In this reanalysis, Spearman first reduced the number of indicators by aggregating tests that he considered to be similar and only then applied two-factor theory and concluded that the general factor from two-factor theory explained almost all of the correlations in the data between the test aggregates. This approach likely inspired Thurstone to develop the higher-order model by extracting a second-order factor from the correlations of his oblique factors in his later work (Thurstone & Thurstone, 1941). The higher-order model can thus be seen as a compromise between Thurstone's work and Spearman's original idea that the shared variance between a battery of intelligence tests is caused by a general factor, or g (Jensen & Weng, 1994; Lang, Kersting, Hülsheger, & Lang, 2010).

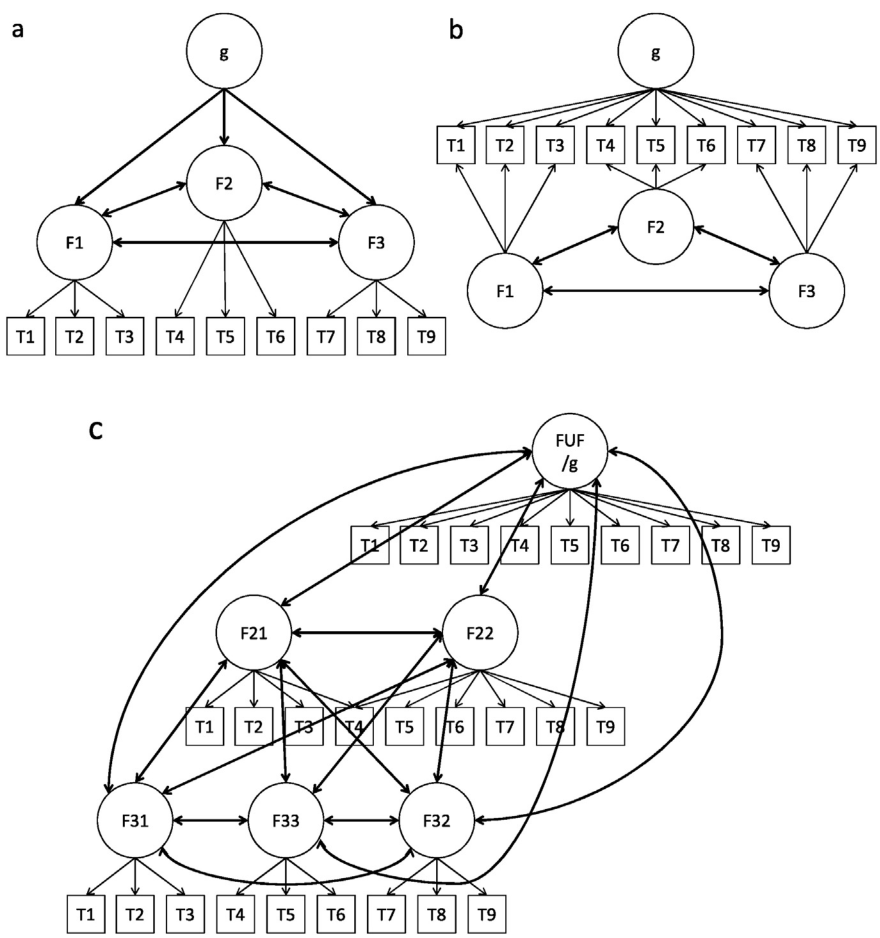

Fig. 1. The higher-order model and the bi-factor model are shown in (a) and (b), respectively. These models seek to find the most parsimonious hierarchical solution. In contrast, the hierarchies of factor solutions approach shown in (c) separately extracts solutions with different numbers of factors from the same indicators and then studies correlations between these factors and represents a different analytical strategy. FUF = first unrotated factor.

A limitation of the higher-order model is the fact that the possible levels of hierarchy is restricted in practice. The reason is that higher-order factors are only extracted based on variance at the hierarchical level just below the level of interest (cf. Fig. 1a). In practice, this characteristic of the model restricts the possible number of hierarchical structures. Carroll (1993), who conducted a large and comprehensive reanalysis of datasets from the intelligence literature using the higher-order model, noted: "One is fortunate to obtain as many as three second-order factors, and this is the minimum number required to support an analysis for a single factor at the third order." (pp. 579).

1.2. The bi-factor model

The bi-factor model (Holzinger & Harman, 1938; Holzinger & Swineford, 1937)—also known as the nested-factors model (Mulaik & Quartetti, 1997) or the hierarchical model (Yung et al., 1999)—offers a second perspective on the status of broad and narrower abilities that is frequently employed in the intelligence literature. This bi-factor conceptualization is distinct from the higher-order model and builds on the idea that broad and narrower abilities only differ in breadth and not in subordination (Humphreys, 1981). The term "bi-factor" derives from the idea that each measurement indicator (i.e., the tests) has more than one (typically two but more is possible, cf. Mulaik & Quartetti, 1997) direct loadings—typically on a general and a narrow factor. Fig. 1b illustrates this conceptualization of the bi-factor model. Fig. 1b also shows that there is no correlation or direct path between the broad factor and the narrower abilities such that these factors are disjunct (i.e., orthogonal). Some methods employed in research using the bi-factor model explicitly require an orthogonal general factor (Jennrich & Bentler, 2012). Other methods do not explicitly require the general factor to be orthogonal but in practice estimation with a general factor that is not orthogonal to other factors is difficult (Mulaik & Quartetti, 1997; Rindskopf & Rose, 1988) such that an orthogonal general factor is used in all research that we are aware of.

The fact that the hierarchical levels are disjunct in the model effectively divides the shared variance between the broad and the narrower factors to either the broad or the narrower factors such that not all shared variance between the general factor and the narrower abilities in the model is attributed to the general factor. The bi-factor perspective consequently does not make the assumption that the general factor has a causal effect on the narrower cognitive abilities. Instead, the broad and narrower factors differ only in breadth (and not in subordination like in the higher-order model). The general factor consequently has shared variance with the test itself that it does not also share with the narrower abilities.

A limitation of the bi-factor model is the fact that the model effectively eliminates shared variance between g and the narrower-factors by orthogonalizing the levels. Removing shared variance through orthogonalization may frequently be desirable in applications (Carroll, 1993), and was especially desirable in studies on predictive validity before methodological techniques were developed that can incorporate shared variance between different levels of the hierarchy (Lang et al., 2010; Krumm, Schmidt-Atzert, & Lipnevich, 2014). When researchers study the hierarchical structure of a construct, however, it may be theoretically desirable to use factor scores that reflect the nature of the data and theoretical assumptions. Orthogonalizing the levels then can be a theoretical limitation as shared variance between the different levels of the hierarchy is a core element of intelligence data (Jensen, 1998; Revelle & Wilt, 2013). In studies on the hierarchical structure of intelligence, orthogonalizing the levels is also an empirical restriction as it effectively limits the number of possible levels to three (commonly only two; cf. Mulaik & Quartetti, 1997).

2. Hierarchies of factor solutions

Research on hierarchies of factor solutions (Goldberg, 2006) is fundamentally based on exploratory factor analysis procedures with which most researchers are familiar. To derive a hierarchy of factor solutions, researchers start by extracting a solution with one (unrotated) factor and by calculating factor scores for this factor. In the next step, a two-factor solution is extracted and rotated, and factor scores as well as the correlations between these factor scores and the factor scores for the initial one factor solution are calculated. The procedure continues by extracting the rotated three factor solution, calculating factor scores for the three-factor solution, and by calculating correlations between the factor scores from the three-factor solution and the factor scores from the two factor solution. This routine continues for subsequent solutions with more factors.

As a refinement of the method, Waller (2007) has shown that it is possible to calculate the correlations between components or factors from different levels of the hierarchy without calculating the corresponding component scores or factor scores. Thereby, problems that might occur when factor scores are to be calculated (Beauducel, 2007) can be avoided (Waller, 2007).

In theory, a hierarchy of factor solutions allows for a number of factorial representations that is equal to the number of variables. However, because the major goal of factor analysis is to develop an adequate and parsimonious description of a domain of interest, the procedure will commonly stop much earlier.

One basic criterion for stopping is when no variables have their highest loadings on a factor. In this case, Goldberg (2006) advised researchers to stop at the level above that one. The reason for this recommendation is that factors without primary loadings are difficult to interpret and are commonly considered to be residual factors.

A second possible criterion for stopping is to rely on the standard approach used in exploratory factor analysis and examine the eigenvalue plot. In examining eigenvalue plots, the goal is to determine how many factors in a given covariance matrix are meaningfully different from random noise. The most common tools for evaluating eigenvalues are Cattell's graphical scree test (Cattell, 1966) and J. L. Horn's parallel analysis procedure (Horn, 1965b). The graphical scree test suggests that eigenvalue plots typically contain a marked decrease or cliff between the meaningful factors and the random factors. Horn's parallel analysis procedure simulates random eigenvalues using the properties of the data so that one can compare the actual eigenvalues with random eigenvalues.

A third criterion for stopping can be used when researchers simultaneously examine several datasets in the same research domain. A criterion for stopping in this context is when a solution does not replicate across datasets to a notable degree. Researchers can then confidentially assume that dataset-specific characteristics dominate in the factor solutions. For instance, research on the structure of personality has yielded no evidence that factors from solutions with more than six factors constantly replicate across languages and samples (e.g., Ashton, Lee, & Goldberg, 2004; Ashton, Lee, Perugini et al., 2004). Most researchers studying several datasets in the same research domain examine eigenvalue plots as a first rough indication on the number of potentially meaningful factors in a dataset. The eigenvalue information, however, is rarely used to ultimately decide on the appropriateness of a taxonomy. For instance, in an analysis of 1710 English personality adjectives or items, Ashton, Lee, and Goldberg (2004) decided to extract five or six factors even though the eigenvalue plot suggested the extraction of seven factors. Ashton, Lee, and Goldberg based this decision on the substantive interpretation of the solutions they extracted and the fact that solutions in other languages supported either five or six factors.

Hierarchies of factor solutions can be studied using either orthogonal or oblique rotations and either using principal components or factor analysis procedures (Goldberg, 2006; Waller, 2007). In the context of personality research, researchers commonly use principal components and orthogonal rotation procedures yielding uncorrelated components at each level of the hierarchy (Goldberg, 2006). The use of orthogonal rotations is based on the observation that many personality dimensions show only small intercorrelations. For the purpose of intelligence research, we suggest using oblique rotations because correlations between factors are likely important in intelligence research. There is strong evidence that intelligence measures are commonly correlated—frequently to a considerable degree (e.g., Jensen, 1998; Revelle & Wilt, 2013). In addition, we propose that factor analysis instead of principal components is generally more appropriate because factor analysis takes measurement error into account (Gorsuch, 1983) and this may be relatively more important when indicators are correlated. Researchers have long used principal components instead of factor analysis for estimating hierarchies of factor solutions because the estimation of correlations between factors of solutions with different numbers of factors had long been problematic statistically (e.g. Grice, 2001). As already noted, Waller (2007) has recently solved this issue by showing how researchers can estimate the correlation matrix between the oblique latent factors for solutions at two different levels of a hierarchy of factor solutions using the rotation matrices for the two levels (see p. 749). Waller's procedure allows researchers to estimate the correlations between the true latent factors and does not require them to use approximations of the true correlation matrix calculated from factor scores (for principal components, the matrix estimated using the rotation matrices and Waller's procedure is identical to the matrix estimated using component scores cf. Grice, 2001).

There are important differences between the solutions-hierarchy approach and the two types of hierarchical models. A potential advantage of the solutions-hierarchy approach is that the extraction of factors at the different hierarchical levels is not influenced by decisions of the researcher regarding the extraction of factors at the other hierarchical levels. The reason is that the extraction of factors at each level is directly based on the manifest indicator variables (personality adjectives or personality-related statements in personality research and cognitive ability tests in cognitive ability research). In contrast, in the higher-order and bi-factor models, the extraction of broader factors is influenced by decisions on extraction of narrower factors. In the higher-order model, the factors higher in the hierarchy are based on the intercorrelations among the lower order factors and not directly on the manifest variables. Variance that the lower-order factors do not capture can also not be captured by the higher-order factors in the model. In the bi-factor model the extraction of broad factors depends on decisions on the narrower factors because the variance in the manifest variables is distributed among the broad and the narrower factors. Consequently, when the number of narrow factors is reduced in a bi-factor model, some variance that has been represented by the narrow factors before can be represented by the broader factor (and vice versa). Fig. 1c illustrates the nature of hierarchies of factor solutions. As shown in Fig. 1c, all factors are directly estimated based on the measurement indicator variables. Furthermore, all factors are correlated with each other. Consequently, g and the narrower factors share variance with all other factors and the test itself.

In summary, the solutions-hierarchy approach has reached considerable sophistication and when used with oblique rotations it allows researchers to not only estimate a hierarchical structure with correlations between factors at different levels of the hierarchy but also between factors within each level of the hierarchy. The solutions-hierarchy approach avoids two specific restrictions of the hierarchical models typically used in intelligence research (orthogonal factors for the bi-factor, and the assumptions of causal higher-order factors in the higher-order model).

3. Intelligence taxonomies and previous research syntheses

Existing syntheses of the intelligence literature have relied on higher-order approaches or bi-factor models (e.g., Carroll, 1982, 1993, 2003; French, Ekstrom, & Price, 1963; McGrew, 2009). These overall literature reviews and other more specific studies have led to the development of a variety of intelligence taxonomies. To provide theoretical guidance for our investigation, we conducted a literature review of the cognitive ability literature that allowed us to develop hypotheses and expectations regarding the emergence of the broader factors at each level of the hierarchical structure extracted by the solutions-hierarchy method. The results of this literature review are shown in Table 1. Because we were primarily interested in parsimonious and replicable taxonomies that could be broadly applied to a variety of datasets, we focused our analyses to solutions with a limited number of factors that could realistically be expected to be replicable across different datasets and different sets of variables.

4. The present investigation

The overarching goal of the present investigation was to study the structure of intelligence using hierarchies of factor solutions. We focused on five datasets that were designed to cover a broad and representative range of ability measures. We then estimated hierarchies of factor solutions and studied which factors emerged at each level of the hierarchical structure of the solutions in each dataset. In so doing, we were interested (a) to what degree solutions in the datasets were similar to theoretical ideas in the reviewed cognitive ability literature and (b) to what degree solutions in different datasets were similar to each other. In addition to these research questions, we also studied how factors from solutions at different levels of the hierarchy (different numbers of factors) were related to each other.

5. Method

5.1. Datasets

We searched for datasets that had a sample size larger than 200, included a broad range of intelligence tests, and were originally assembled for the purpose of conducting an exploratory investigation of the general factor structure of intelligence (and not a specific domain of intelligence or cognitive abilities) in literature reviews (Carroll, 1993) and in databases of psychological literature (e.g., PsychINFO, Google Scholar, and WorldCat). This search yielded six datasets that fulfilled these criteria. For one dataset, no correlation matrices or raw data could be obtained (Jäger, 1967), and for one dataset only a correlation matrix of composite scores combining several cognitive abilities was available (Hakstian & Cattell, 1974). For four datasets, a product–moment correlation or covariance matrix of the cognitive ability tests was available. These datasets were THUR41 (Thurstone & Thurstone, 1941), HORN65 (Horn, 1965b), SCHO76 (Scholl, 1976), and WOTH (Wothke et al., 1990).

We additionally had access to an extended version of a previously published dataset (BEAU02; Beauducel & Kersting, 2002). This extended dataset included the published data as well as additional data. The additional data included two additional tests, and 7303 additional participants such that the study population included a total of 16,823 persons. The two additional tests were a dictation test assessing the ability to write down spoken text without orthographic errors, and a clerical work-sample test asking participants to determine postal rates based on a couple of different information parameters.

Table 2 provides details on the five datasets. We do not suggest that these datasets cover the universe of possible data sets. However, the datasets are a relevant subset of the relevant intelligence literature allowing for substantial investigation of the unrestricted nested-factors model.

Table 1
Some common ideas on factor taxonomies at different levels of the intelligence hierarchy.

Factors	Model/author(s)	Prediction
1	Jensen, 1998; Lubinski, 2004; Vernon, 1988	Explains 50% of the variance
1	Gustafsson, 1984; Jensen, 1998; Undheim & Gustafsson, 1987; Spearman, 1904	Fluid tasks like Raven's progressive matrices have the highest loadings.
1	Robinson (1999, 2005)	Knowledge and acquired skills have the highest loadings
1	Ashton, Lee, and Vernon (2001, 2005)	Fluid and crystallized intelligence show similar loadings
2	P. E. Vernon's hierarchical theory of intelligence (Vernon, 1950)	v:ed (verbal:educational) captures abilities like verbal ability, numerical facility, logical reasoning and fluency perceptual factor labeled k:m (spatial:mechanical) that is dominated by spatial abilities, mechanical information, psychomotor coordination, reaction times and manual skills
2	gf–gc theory (Cattell, 1943, 1963, Horn, 1965a, 1976	general crystallized factor (gc) capturing the ability to use knowledge, acquired skills, and experience as well as a general fluid factor (gf) tapping the ability to think logically and solve problems in novel situations.
3	Gustafsson's HILI model (Gustafsson, 1984)	gf–gc and a visualization factor (gv) that includes all tasks with figural content.
3	Verbal-perceptual-image rotation model (Johnson & Bouchard, 2005)	Verbal and perceptual factors similar to Vernon's hierarchical theory, additional image rotation factor
3	Figural-verbal-numerical taxonomy (Ackerman, 1988; Ackerman, Beier, & Boyle, 2005; Marshalek, Lohman, & Snow, 1983), also included structure of intellect model (Guilford, 1967) and the Berlin model of intelligence structure (Süß & Beauducel, 2005)	Figural, verbal, and numeric content loads on three separate factors
4	Ackerman's modified version of the radex model (Ackerman, 1988; Ackerman et al., 2005)	figural, verbal, and numerical power abilities and more basic speed abilities
4	Operation facets of the Berlin model of intelligence structure (Beauducel & Kersting, 2002; Jäger, Süß, & Beauducel, 1997; Süß & Beauducel, 2005)	a broad reasoning factor that primarily captures the ability to think logically, a memory factor, a fluid ability factor that taps the ability to generate diverse ideas and material like words or figural shapes, and a speed factor that captures the ability to process material and quickly solve simple tasks.
5	Extended gf–gc theory (Hakstian & Cattell, 1974; Horn, 1965a; Horn & Cattell, 1966)	gf–gc and additionally: general cognitive speed (gs), visualization capacity (gv) that taps the ability to integrate and organize visual material, general memory capacity (gm), and general retrieval capacity (gr) that captures the ability to retrieve information from long-term memory and is largely similar to the concept of fluency in other intelligence models. A notable difference, however, is that verbal and word fluency factors are assigned to gc and not to the gr dimension in the theory so that that gr is not perfectly identical to fluency constructs in other frameworks.
7	Three stratum theory of cognitive abilities (Carroll, 1993)	Second stratum includes gc, gf, gv, gs, and gr from the extended gf–gc theory and additionally two new factors: Auditory perception and reaction time decision speed
8	Primary mental abilities (Thurstone, 1938a, Thurstone & Thurstone, 1941)	Word fluency, verbal comprehension, spatial visualization, number facility, associative memory, reasoning, and perceptual speed

5.2. Analytical strategy and statistical analyses

We examined the hierarchy of factor solutions using the procedures described in Goldberg (2006) and Waller (2007). As discussed previously, we relied on factor analysis and oblique rotation. Specifically, we used principal axis factor analysis. All solutions with more than one factor were promax-rotated ($m = 3$).

To study the relations between the solutions at the different levels of each dataset, we estimated the correlations between the factors using the procedures described in Waller (2007). We therefore relied on a modified version of Waller's (2007) syntax for the R programming environment (R Development Core Team, 2010). Waller's program was originally developed for orthogonal (varimax) rotation. We therefore modified the program for the use with promax rotation and factor analysis. Subsequently, we graphed the findings by translating the results of the solutions-hierarchy approach into a graph in the dot language (AT&T Labs Research and Contributors, 2011).

We extracted solutions until the majority of the factors did not replicate across datasets anymore. Although we relied on the replication criterion, we also obtained eigenvalues for all five datasets as useful descriptive information on the number of potentially meaningful factors in the samples and as evidence on how dominant the first factor (g) was in each of the matrices (see, e.g., Ashton, Lee and Goldberg, 2004; Ashton, Lee, Perugini, et al., 2004). To foster the interpretation of the eigenvalues, we examined Cattell's graphical scree test (Cattell, 1966) and J. L. Horn's parallel analysis procedure (Horn, 1965b). The parallel analysis procedure simulates random eigenvalues using the properties of the data so that one can compare the actual eigenvalues with random eigenvalues.

6. Results

6.1. Eigenvalues and scree plots

Fig. 2 provides eigenvalues and simulated random eigenvalues from the parallel analysis procedure for all five datasets. As indicated by Fig. 2, there was evidence for a strong first factor in all eigenvalue plots.

The total number of factors to extract based on the graphical scree criterion (first marked eigenvalue increase) differed considerably across the five datasets. The scree criterion suggested that there were three or five factors in the THUR41 data, four factors in the HORN65 data, four factors in the SCHO76 data, two or six factors in the BEAU02 data, and two, three, or four factors in the WOTH90 data.

The number of extracted factors also differed when we used parallel analysis instead of the graphical scree procedure. As indicated by Fig. 2, the simulated random eigenvalues derived by the parallel analysis procedure suggested that there were four factors in the THUR41, SCHO76, and WOTH90 datasets, three factors in the HORN65 data, and seven factors in the BEAU02 dataset.

6.2. Hierarchy of intelligence factor solutions

Figs. 3 to 7 provide the graphical summaries of the solution-hierarchy analyses and show content descriptions as well as correlations between the factors at adjacent levels of the factor hierarchies for each dataset. In the graphs, the boxes represent the factors. The two-digit factor numbers refer to the overall number of factors in the solution from which the factor is derived (first digit), and the number of the factor in the solution (second digit). Correlations higher than

Table 2
Datasets included in the review.

Label	Author(s) and year of publication	Sample		Material	
		N	Characteristics	No.	Source(s)
THUR41	Thurstone and Thurstone (1941)	710	Elementary school children,	60	New developments; tests from previous publications by L. L. Thurstone (1938a, 1938b)
HORN65	J. L. Horn (1965a)	297	Prisoners, school children, and unemployed persons, mean age: 27.6 years (SD = 10.6; Range = 14–61); 27.61% female	38[b]	7 tests taken from Botzum (1951) 5 tests taken from previous research by R. B. Cattell 2 tests from previous research by J. P. Guilford 3 tests from previous research by L. L. Thurstone 1 test taken from Taylor (1947)
SCHO76	Scholl (1976)	276	German police trainees, mean age: 20.4 years (Range = 16–36)	63	57 translations of tests from the 1963 Kit of Factor-Referenced Tests (French, Ekstrom, & Price, 1963); 3 newly developed tests; 2 tests from the Leistungsprüfsystem (LPS; W. Horn, 1962); 1 test from the Intelligenzstrukturtest (I-S-T; Amthauer, 1955)
WOTH90	Wothke et al. (1990)	6751 overall but participants worked on different booklets, *N*s for each correlation vary between 207 and 701; harmonic mean 230	Air-force recruits, 16.9% female	56	Armed Services Vocational Aptitude Battery (ASVAB, 10 tests); 46 tests from the 1976 Kit of Factor-Referenced Tests (Ekstrom, French, Harman, & Derman, 1976)
BEAU02	Beauducel and Kersting (2002)[a]	16,823	Applicants for governmental positions in Germany, Mean age: 21.39 (SD = 4.41; Range = 15–57); 58.10% female	44	New developments; fluid tasks are based on previous work by Jäger et al. (1997)

[a] The dataset we analyzed in the current article is an extended version of the dataset analyzed in the 2002 publication.
[b] Additionally includes six tests measuring attention and one test that was highly similar to another test (both were mechanical knowledge tests and correlated at r = .69). Because attention is considered to be related to intelligence but is treated as a separate construct in the literature (e.g., de Jong & Das-Smaal, 1995; Schweizer, Moosbrugger, & Goldhammer, 2005) and because the parallel tests yielded a strong test-specific factor, these tests were excluded.

.50 are shown with solid lines. For factors having no correlations higher than .50 to the next level, the highest correlation is shown using a dashed line.

6.2.1. First-unrotated factor

We started by examining the content of the first unrotated factors (FUF). These analyses revealed that tests with high loadings did not

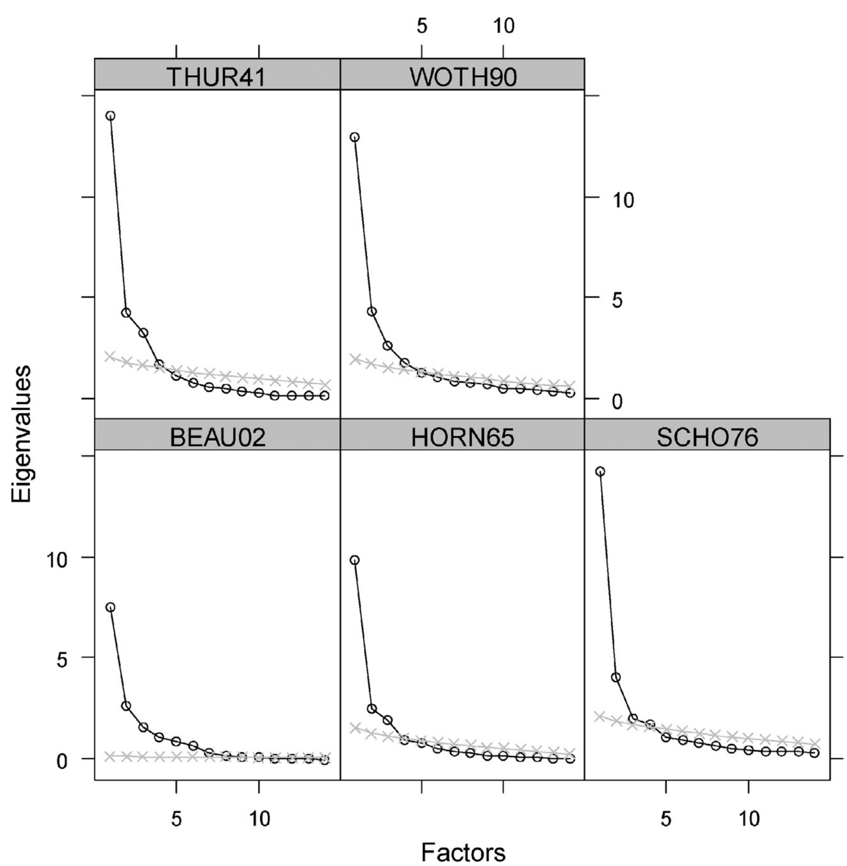

Fig. 2. First fifteen eigenvalues (o) and first fifteen simulated random eigenvalues from Horn's parallel analysis procedure (x) for the five datasets included in the present study.

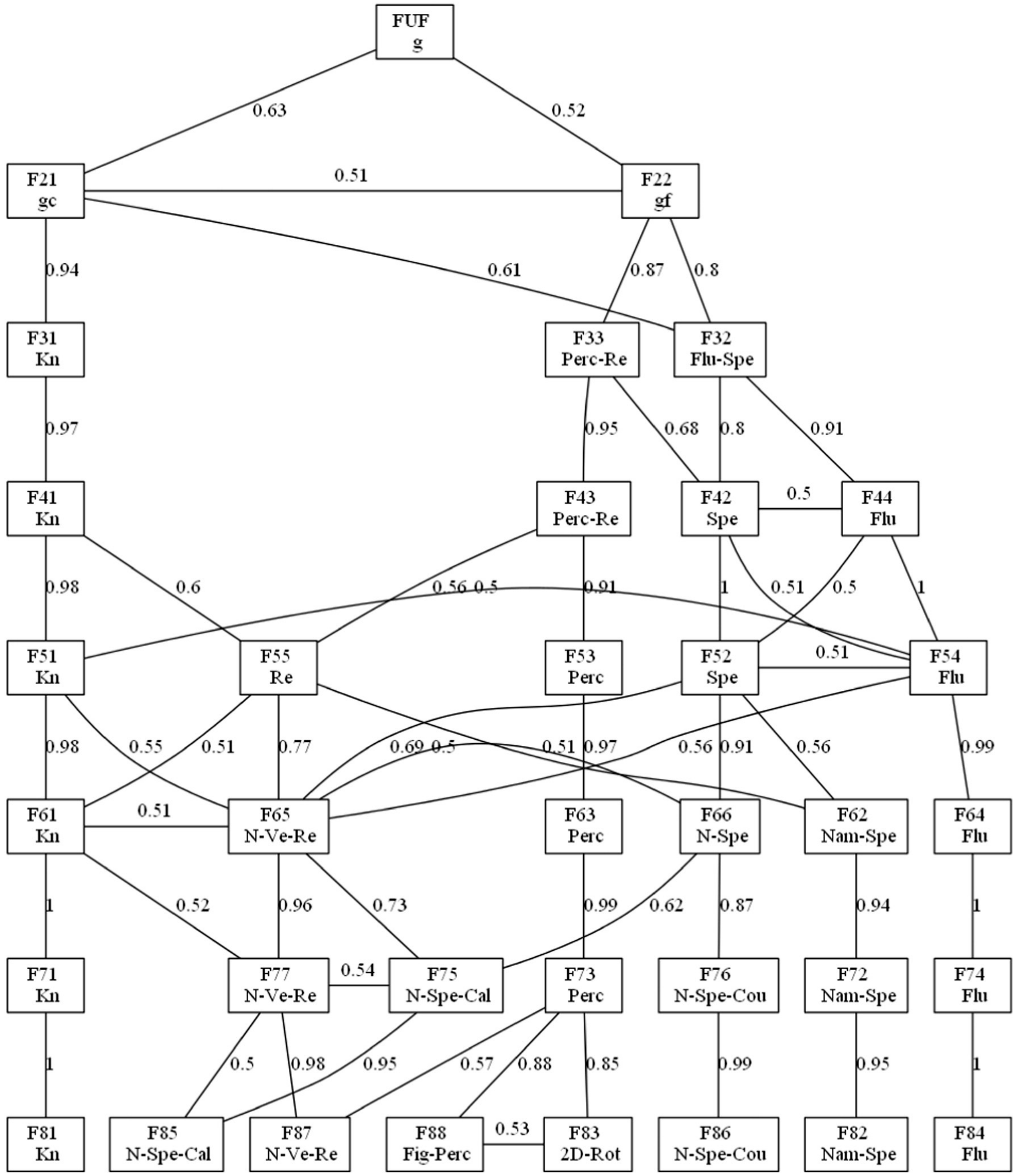

Fig. 3. The hierarchical structure of the Thurstone and Thurstone (1941) dataset.

only included tests commonly considered to be good indicators of g in the literature but also speed and fluency tests. The FUF was consequently quite diverse and broad in our datasets featuring large and diverse samples of cognitive ability tests.

6.2.2. Two-factor solutions

At the second level of hierarchy, three of the five datasets (THUR41, HORN65, and BEAU02) showed two factors which were reminiscent of a classic gf–gc structure with a general crystallized factor (gc) capturing acquired skills, knowledge, and experience as well as a general fluid factor (gf) tapping logical thinking and problem-solving abilities in novel situations. The gf and gc factors all showed considerable correlations with the FUF or g (see Figs. 3 to 7). In one dataset, gc had a stronger relation to g than gf (THUR41). In the two other datasets, gf was more strongly related to g than gc (HORN65, BEAU02). The relationships between g and gf as well as g and gc were frequently substantial but there was no correlation higher than $r = .74$.

The two datasets that did not clearly find a classic gf–gc structure showed different deviations from the gc–gf structure. In the SCHO76 data, the deviations from the gf–gc structure were relatively modest. In this dataset, verbal fluency tasks (Flu) combined with knowledge-related tasks (Kn) to form a strongly verbally-characterized gc factor. The assignment of verbal-fluency tasks on gc has been proposed in the literature before. For instance, Carroll's three-stratum theory (1993) conceptualizes verbal fluency as a subcomponent of gc. The underlying idea is that verbal fluency tasks commonly require that person's retrieve a substantive variety of vocabulary from (crystallized) long-term memory. However, in the SCHO76 data, also fluency tasks that require only very basic processing of verbal material load on the gc factor so that the combined factor in this datasets is broader than the gc factor

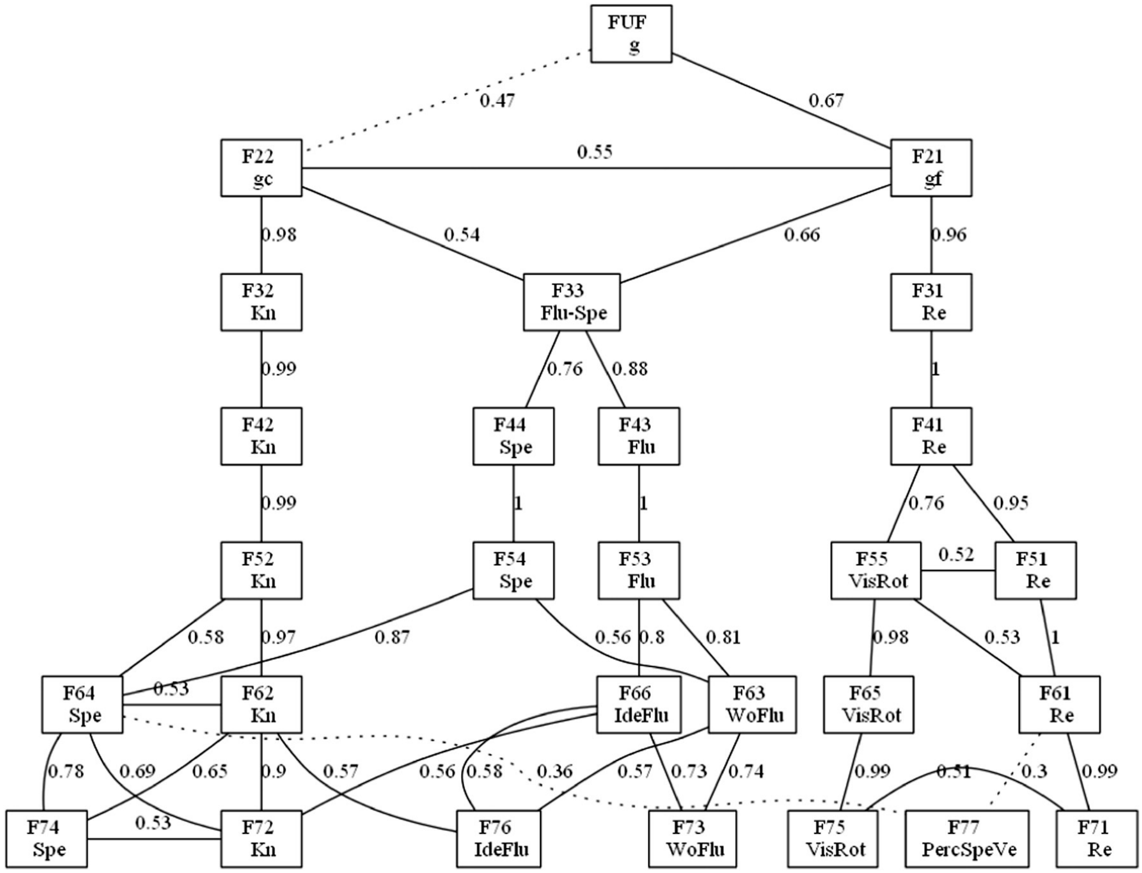

Fig. 4. The hierarchical structure of the Horn (1965a) dataset.

expected by gf–gc theory and its modifications (e.g., Hakstian & Cattell, 1974; Horn & Cattell, 1966).

The deviations from the gc–gf structure in the other dataset, the WOTH90 data, were more substantial. In this dataset, one factor captured all types of strongly speeded tasks including simple speeded fluency tests (Spe-Flu), and the other factor included all less speeded reasoning (Re) and knowledge (Kn) tasks. This structure is most reminiscent of Ackerman's (1988) differentiation between level/power and speed abilities in his extended version of Guttmann's radex model (Guttman, 1965). Overall, it should be noted that there are substantial differences between the gc–gc factors in the different data sets and that also the correlation of gf, gc, and g shows considerable variation across data sets.

6.2.3. Three-factor solutions

There was considerable agreement on the third level of the hierarchy. In all five datasets, a basic three-factor structure emerged that consisted of one factor with basic speed abilities (Spe), one factor with reasoning tasks (Re), and one factor with knowledge tasks (Kn). The speed factor was characterized by tasks that require people to perform simple overlearned operations quickly. The reasoning factor was characterized by tasks that require the deep processing of complex figural, verbal, and numerical tasks. Finally, the knowledge factor contained tasks that asked for factual knowledge from long-term memory and was very closely related to the gc factors from the two-factor solutions but did not contain verbal tasks that also required reasoning. Although we found this basic structure in all five datasets, there were nevertheless differences in the exact nature of these factors. Especially, the datasets differed in how fluency (Flu), memory (Me), and perceptual speed tasks (Perc) were assigned to the three factors.

6.2.4. Four-factor solutions

At the fourth level, the speed-reasoning-knowledge taxonomy from the third level remained intact in all solutions. The new fourth factor that emerged differed between the datasets. In three datasets (HORN65, THUR41, and WOTH90), a new separate fluency factor emerged. In the other two datasets (SCHO76, and BEAU02), the four-factor solution included a separate memory factor in addition to the speed-reasoning-knowledge taxonomy.

6.2.5. Five-factor solutions

At the fifth level, four datasets included four core factors: Speed, reasoning, knowledge, and fluency. The exception was the SCHO76 data in which knowledge and fluency could not be separated. In addition, there was evidence for a memory-perception factor. The empirical picture for this factor was more difficult to interpret because not all datasets included a considerable range of memory and perception tasks. In two datasets (SCHO76, BEAU02), the memory factors from the four-factor solutions continued to exist. These datasets contained a range of memory tasks but no prototypical perceptual tasks. The perceptual tasks included in these two datasets involved either speed or reasoning and consequently loaded on these factors. In one dataset (THUR41), a perception factor emerged. This dataset included a limited number of memory tasks but a considerable range of perceptual tasks that were not primarily characterized by speed or reasoning. Finally, in the only dataset containing both a considerable range of memory and perception tasks, we found a combined perception-memory factor at the fifth level (WOTH90).

6.2.6. Six, seven, and eight factors

While the present set of analyses suggests that there is considerable agreement at the first five levels of the solution hierarchy, we found

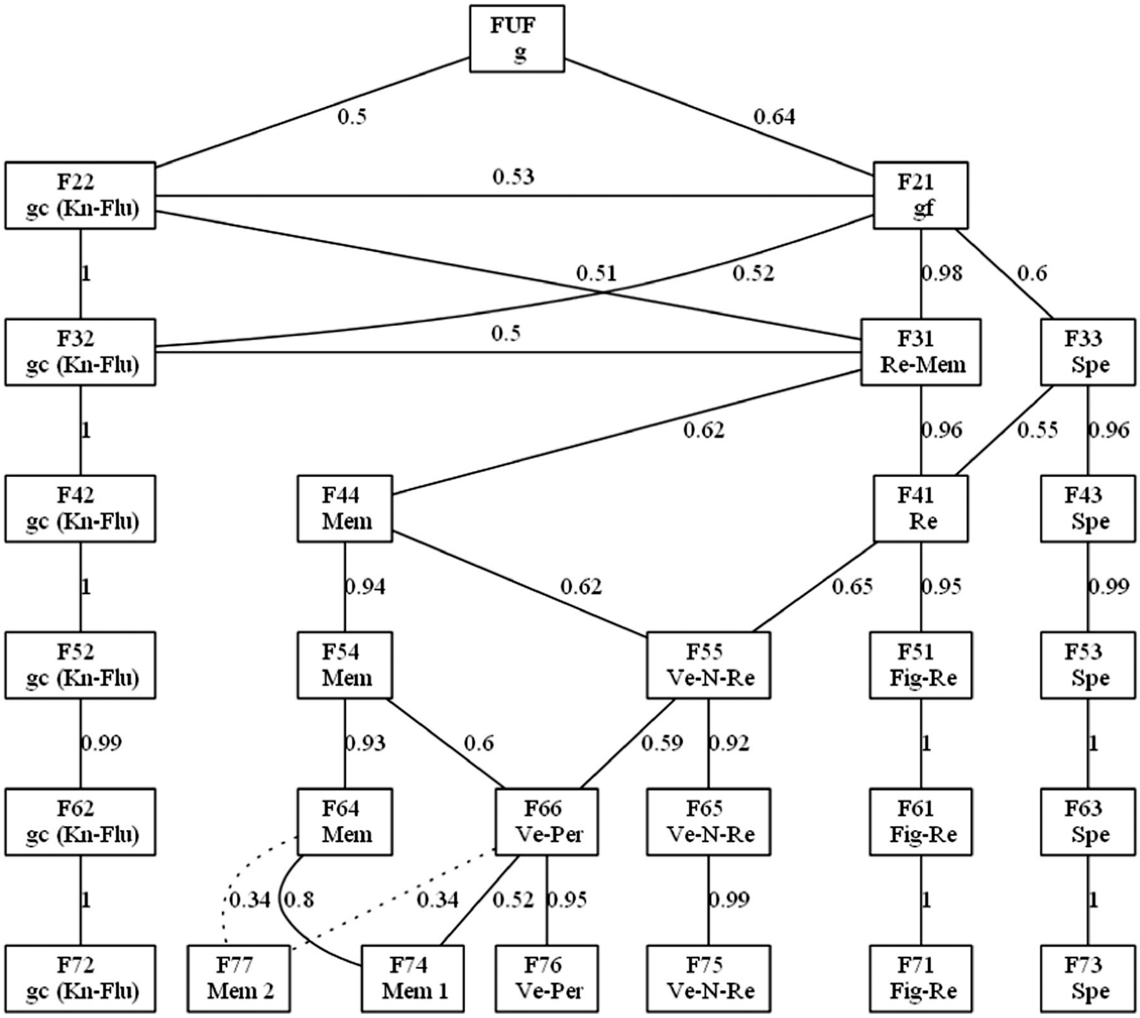

Fig. 5. The hierarchical structure of the Scholl (1976) dataset.

considerably less convergence at subsequent levels and a variety of specific factors. These factors included combinations of naming speed (Nam-Spe), calculation (Cal), 2D rotation (2D-Rot), visual rotation in general (VisRot), counting (Cou), word fluency (WoFlu), ideational fluency (IdeFlu), general ideational (Id), and mathematical (Math) abilities.

The only finding that occurred in more than one dataset was that the reasoning factor splitted into content-material specific subfactors. This type of split occurred in three of the five datasets (THUR41, SCHO76, and BEAU02). In two of these three datasets (SCHO76 and BEAU02), the resulting content-specific reasoning factors largely consisted of verbal, numerical, and figural material and were thus in correspondence with the figural-verbal-numerical taxonomy (e.g., Ackerman, 1988; Guilford, 1967).

7. Discussion

In this article, we built on personality research and suggested that studying hierarchies of factor solutions can also be useful for research on the structure of intelligence. Our reanalysis yielded new insights into the nature of the intelligence construct by identifying some structural consistencies at the top of the hierarchy of factor solutions that were reasonably stable across the five datasets. In this discussion section, we elaborate both on the substantive findings of our investigation as well as the methodological implications of studying hierarchies of factor solutions in intelligence research. We begin by discussing the substantive findings.

7.1. Hierarchies of intelligence factor solutions

The present investigation focused on five datasets on the general structure of intelligence. Fig. 8 provides a summary of our findings at the first five level of the hierarchy of factor solutions in these five datasets.

As Fig. 8 illustrates, one finding of our investigation was that we found evidence for the gc–gf distinction proposed by Cattell and J. L. Horn (Cattell, 1943, 1963; Horn, 1965a, 1976) at the second level of the hierarchy. A gf factor emerged in four, and a gc factor emerged in three of the five datasets. There were some differences in the relations between the gc and gf factors at the second level and the g factor. Although one of these two relations was typically high in each of the datasets, none of these correlations exceeded $r = .74$. Our analyses consequently suggest that both gc and gf carry meaning that is different from the g factor at the top in hierarchies of factor solutions.

A second major finding was that there was a factor structure with a reasoning, a speed, and a knowledge factor at the third level of the solutions hierarchy. This basic structure was present in all five datasets (see Fig. 8) even though the exact nature of the factors somewhat differed across the five datasets. The fact that we found this basic taxonomy in all five datasets is remarkable given that the datasets considerably varied in the used material, context of the data collection (personnel selection vs. participation in research), time (1940s to the 2000s), and sample characteristics. The reasoning-speed-knowledge taxonomy is not identical to a specific model in the previous intelligence

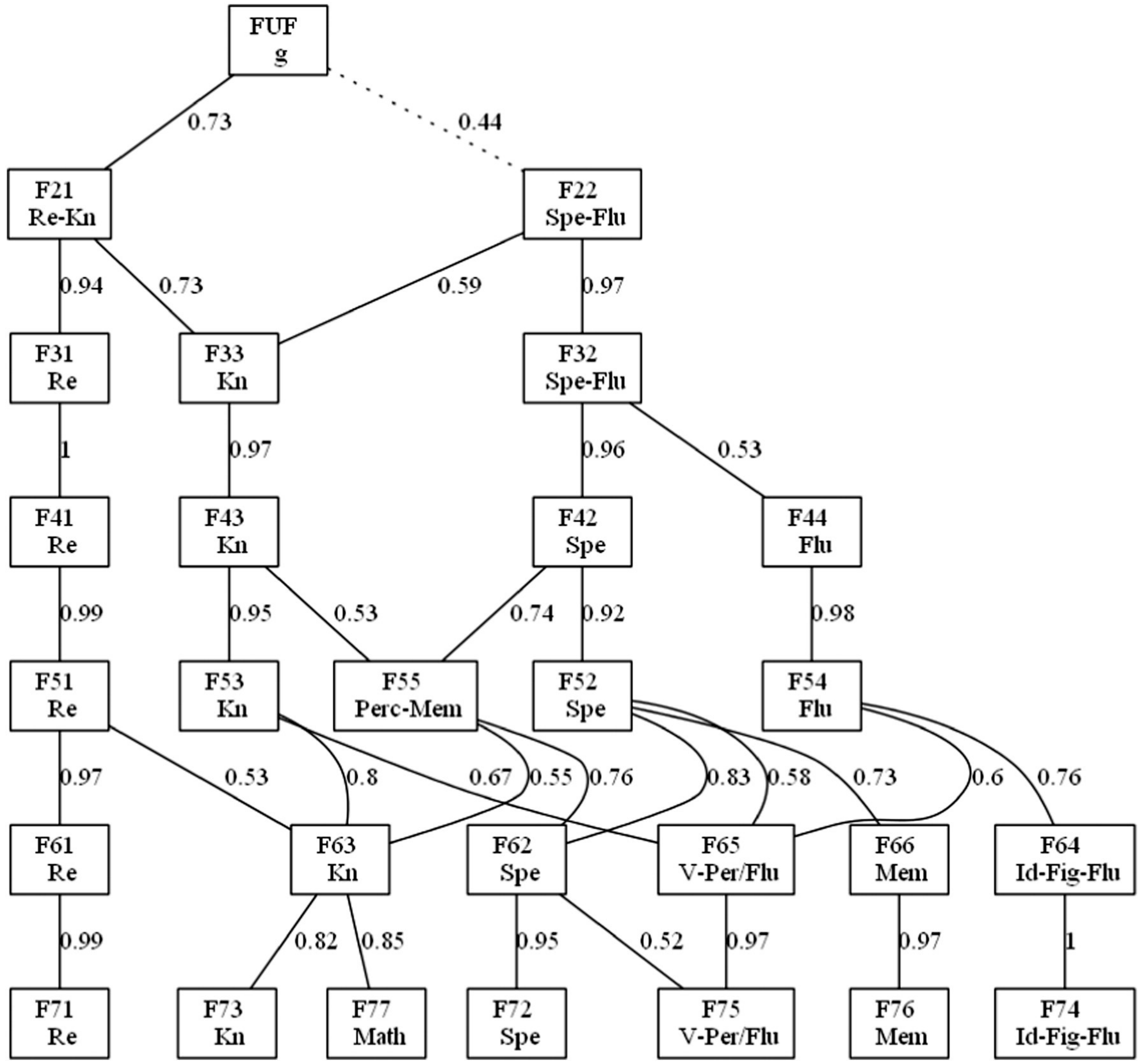

Fig. 6. The hierarchical structure of the Wothke et al. (1990) dataset.

literature but nevertheless shows considerable similarities to models previously discussed in the literature. Specifically, the distinction between speed and reasoning abilities can also be found in the radex model of intelligence (e.g., Marshalek et al., 1983). However, both speed and knowledge tasks can be classified into the rule-application segment of the radex model (Marshalek et al., 1983). Moreover, gc has been located in the rule-application segment of the radex model (Marshalek et al., 1983), so that the radex model does not distinguish between gc, knowledge, and speed. It should be noted that the radex model could only be calculated by means of smallest space analysis (Guttman, 1968), which leads to a representation of the data that can usually not be found by means of factor analysis. Furthermore, the distinction between reasoning and speed abilities is also a key building block of the Berlin model of Intelligence structure (Beauducel & Kersting, 2002; Jäger et al., 1997; Schulze, 2005; Süß & Beauducel, 2005). However, this model does not consider knowledge abilities. The reason for the fact that we did not find content factors for verbal, numerical, and figural abilities at the third level even though these factors have frequently been documented in the literature (Ackerman, 1988; Jäger et al., 1997; Marshalek et al., 1983) could also be related to the fact that the solutions-hierarchy approach aims at finding the most robust structures in the data. In contrast, smallest space analysis and related faceted confirmatory factor models (Süß & Beauducel, 2005) are aimed to identify complex overlapping structures.

The third major finding was that we found considerable agreement on the fourth and fifth level of the hierarchy (see Fig. 8). Specifically, we found evidence for a taxonomy consisting of factors for speed (5 of 5 datasets), reasoning (5 of 5 datasets), fluency (4 of 5 datasets), and knowledge (4 of 5 datasets). In addition to this speed-reasoning-fluency-knowledge taxonomy, there was also some evidence for the existence of a memory-perception factor (3 datasets). We speculate that this factor was difficult to identify because some of the datasets did not contain a sufficient number of both prototypical perception and prototypical memory tasks. Accordingly, a factor primarily characterized by memory tasks emerged in two datasets (SCHO76, BEAU02), and a perception factor emerged in one dataset with a small number of memory tests (THUR41). In the dataset that includes a variety of both types of tasks (WOTH90), an integrated factor emerged.

In summary, this study suggests that intelligence as operationalized in the five datasets that we studied can be characterized by a 1–2–3–5 hierarchy of factor solutions with a g factor at the top of the solutions hierarchy, gc and gf factors at the second level, a speed-reasoning-knowledge taxonomy at the third level, and a four- or five-factor taxonomy with a speed, a reasoning, a fluency, a knowledge and possibly a memory-perception factor at the fourth or fifth level. The evidence for the speed-reasoning-knowledge taxonomy at the third level was somewhat stronger than the evidence for the gc/gf and the speed-reasoning-fluency-knowledge-memory/perception taxonomy

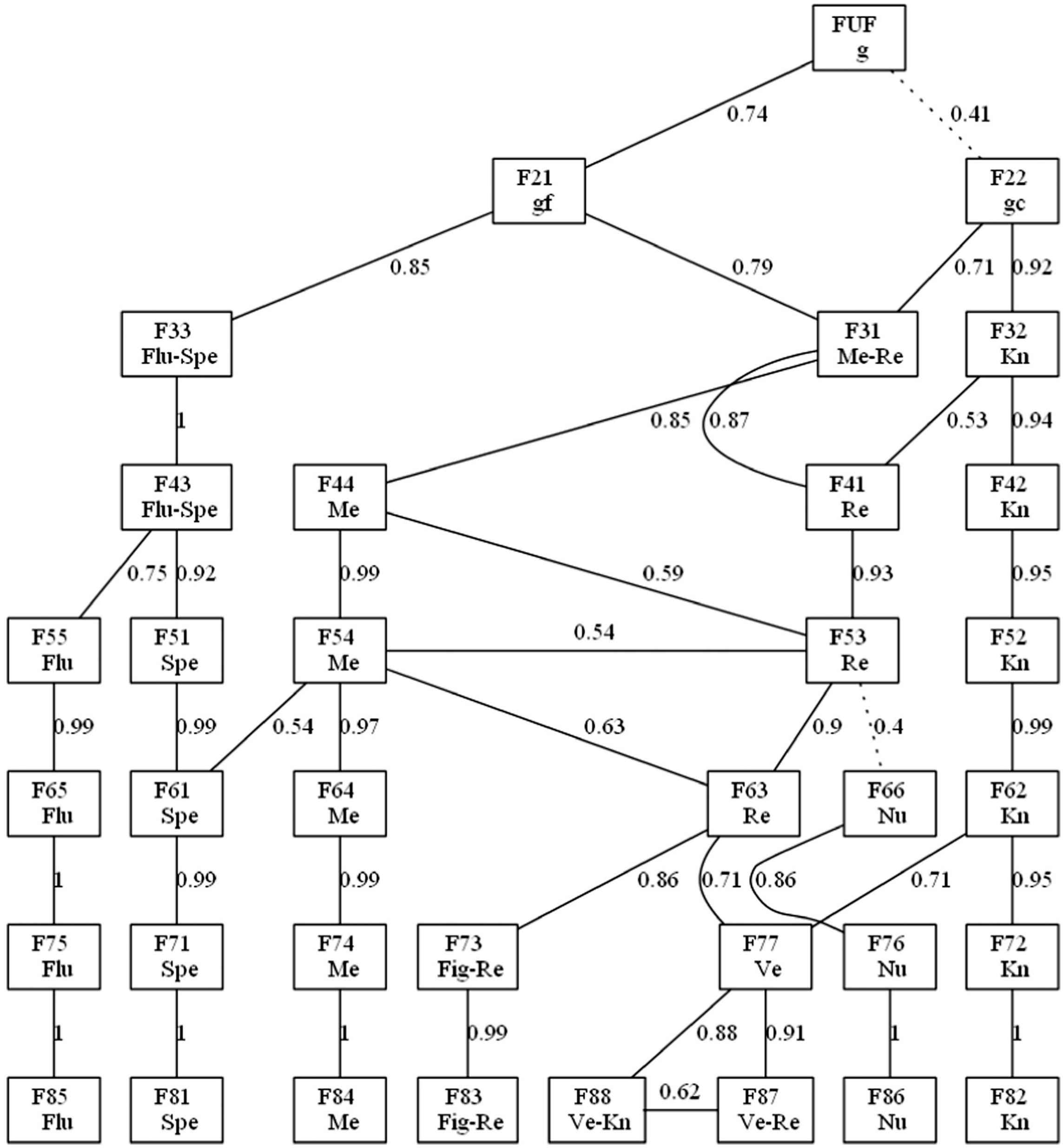

Fig. 7. The hierarchical structure of the Beauducel and Kersting (2002) dataset.

at the fifth level because we found evidence for factors of this type in all five datasets (see Fig. 8). Overall, the findings of this study suggest that intelligence has a relatively stable hierarchy of factor solutions in the five datasets we investigated. The differences between the hierarchies of factor solutions in the five datasets were overall comparable to the differences typically found between investigations in the field of personality. Lexical studies of personality commonly also show considerable agreement but nevertheless also show some deviations across different languages or different sets of items (Ashton, Lee, & Goldberg, 2004; Ashton, Lee, Perugini et al., 2004).

Our study provides researchers, test developers, and practitioners with a different representation of the structure of intelligence. This different representation has several implications for intelligence research.

One implication of our study is that its findings provide a basis for structuring future meta-analyses and reanalyses of intelligence research just like lexical studies studying hierarchies of factor solutions and the five- and six-factor taxonomies were useful for personality researchers (Ashton & Lee, 2005; Saucier & Goldberg, 2001). The 1–2–3–5 taxonomy includes relatively broad factors and can thus be used on datasets that use a variety of different types of intelligence batteries. This characteristic may especially be useful when researchers seek to structure literature on the relationship between different types of intelligence measures and outcome criteria.

A second implication of our study for intelligence research is that it could provide a basis for the development of new intelligence batteries and measures. One possibility would be to develop a battery that captures the entire 1–2–3–5 structure. However, because hierarchies of factor solutions typically include considerable overlap between adjacent levels, we believe that a convenient approach for researchers could be to target one or two specific levels of the 1–2–3–5 taxonomy. Researchers could then try to select a set of intelligence tasks that adequately capture these levels of the taxonomy (e.g., the third or the second and the fifth level). An advantage of this approach is that it is relatively simple. Consequently, there may be a good chance that factor solutions replicate well across different contexts and datasets.

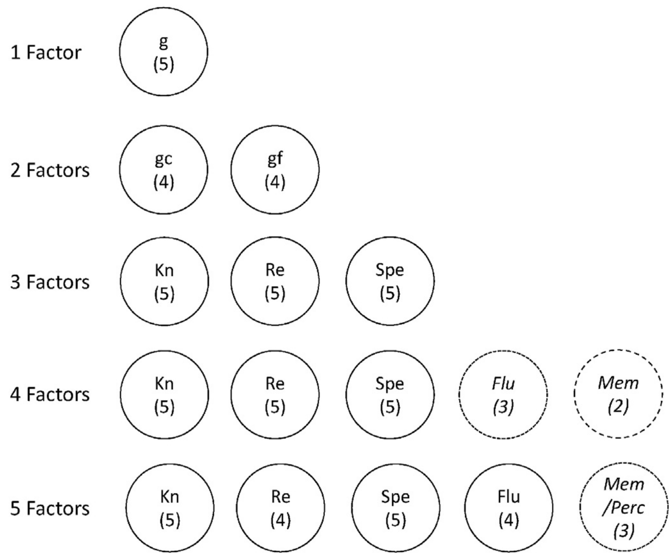

Fig. 8. Overview and summary of the findings. The numbers in parentheses indicate the number of datasets with evidence for the respective factor.

7.2. Methodological implications

Intelligence research has traditionally focused on higher-order factor analysis and bi-factor analysis, and these approaches have contributed to significant progress in the field of intelligence research. A limitation of these approaches is that the extraction of factors at each level is affected by the extraction of factors at other levels. This characteristic limits the number of hierarchical levels in higher-order and bi-factor models to commonly not more than three (cf. Carroll, 1993; Mulaik & Quartetti, 1997). The solutions-hierarchy approach adds a different representation of intelligence. Researchers can use the solutions-hierarchy approach to study relationships between a large number of levels and to graph relationships between closely related factors. The solutions-hierarchy approach may especially be useful when researchers are interested in providing a description of an intelligence dataset and when they seek to identify differences and similarities across datasets from different traditions of intelligence research.

Another way in which the solutions-hierarchy approach may be useful for researchers is when they seek to study correlated factors with different levels of broadness/generality in outcome criteria but do not want to make assumptions on the causal direction of the correlations (Humphreys, 1981; Sternberg, 1981). The unspecified causal direction between the more general and more specific factors in the solutions-hierarchy approach may have practical implications when it comes to studying the role of correlated intelligence constructs in outcome criteria (Lang et al., 2010; Krumm et al., 2014). When researchers assume that a g factor causes the correlations between intelligence tests, the higher-order model and incremental validity analysis are the adequate tools for studying the role of g and narrower intelligence constructs in outcome criteria. In the higher-order model, g has a causal influence on the narrower cognitive abilities (see Fig. 1a) and the higher-order model therefore assumes that all shared variance between g and the narrower cognitive abilities is caused by g. In line with this idea, g should be entered first in incremental validity analyses. In contrast, when researchers wish to not make the a-priori assumption that shared variance results from g, they can base their research on the bi-factor or the solutions-hierarchy approach as both approaches do not make assumptions on the causal order of g and the narrower cognitive abilities. In the solutions-hierarchy approach, shared variance between g and the narrower cognitive abilities can be retained without assumptions on the source of the shared variance. The relationship between the correlated factors from a hierarchy of factor solutions and the criterion can then be studied using regression-based techniques that do not use assumptions on a causal order of the predictors (Lang et al., 2010). Regression-based techniques of this type include relative importance analysis (Grömping, 2007) and dominance analysis (Azen & Budescu, 2003). Studying hierarchies of factor solutions consequently does not only provide a different approach for studying the structure of intelligence but also has implications for studying relationships between intelligence constructs and outcome criteria.

7.3. Limitations

One limitation of our study concerns the source of the differences between datasets. As noted previously, there were considerable similarities in the factor structures from each of the datasets, and these similarities were surprisingly large given that the five datasets considerably varied in the used material, the context (applicants vs. volunteers), the time (1940s to the 2000s), and the characteristics of the participants. Nevertheless, there were also some relevant differences between the datasets. A limitation of our investigation is that it is difficult to tell why these differences occurred and what factor likely caused these differences.

A second limitation is the number of manifest variables in the current analyses. The number of tests in the five datasets ranged from 38 to 63. Although these numbers are considerably higher than the number of tests in most investigations on the structure of intelligence and are the largest available matrices on the general structure of intelligence, these numbers are still considerably smaller than the number of manifest variables in studies examining hierarchies of factor solutions in the field of personality research. The reason is that gathering data on a manifest variable in personality research (a personality item consisting of a Likert-scale rating of a personality-related adjective or statement) consumes less time than gathering data on a manifest variable in intelligence research (a test consisting of multiple similar items). Nevertheless, the factors in the stable taxonomies we identified still were all based on a considerable number of variables. Furthermore, it is likely that the variables in intelligence-structure investigations are considerably more reliable than the single-item ratings used in personality research so that the higher reliability of the variables may partly make up for the lower overall number of variables.

Another limitation of our reanalyses is the fact that the datasets in our investigation consisted of relatively young samples. The intelligence literature suggests that gc and gf develop differently across the lifespan (Cattell, 1963; Horn, 1976) and also that performance in specific tests changes differently across the life span and across cohorts (Skirbekk, Stonawski, Bonsang, & Staudinger, 2013). We accordingly recommend future research using older and more age-diverse samples.

In addition to the described limitations that relate to the nature of the datasets that we reanalyzed, there are also limitations of the solutions-hierarchy method. One limitation of the approach is that it is based on exploratory factor analysis. Exploratory factor analysis is data-driven in its nature and does not directly allow researchers to test the degree to which pre-specified factor models fit a particular dataset.

A second limitation of the solutions-hierarchy approach is that it is typically difficult to objectively judge how many factors should be extracted. Decisions on the number of factors that should be extracted typically depend on the interpretation of the factors especially when researchers study different datasets. This limitation is not specific to the use of hierarchies of factor solutions for the purpose of studying the structure of intelligence.

A third limitation of the solutions-hierarchy approach is that it is primarily suited to describe relationships between intelligence factors. The solutions-hierarchy approach is not designed for testing specific causal theories on how intelligence factors influence each other. For instance, the solutions-hierarchy approach is capable of describing the correlation of g with gc and gf factors at the second level. However, the approach is not capable of, for instance, testing theoretical ideas on effects of g on gc and gf (Undheim & Gustafsson, 1987; Valentin Kvist & Gustafsson, 2008).

Finally, a fourth limitation of the solutions-hierarchy and most other research on the structure of intelligence is the fact that factor models do not necessarily provide insights on the functional level of the human brain (Bartholomew, 2004). In an ideal scenario, researchers would like to find correspondence between intelligence factors identified in research on the structure of intelligence and individual differences in biological characteristics. However, correspondences of this type are not easy to identify and require biological research (Bartholomew, 2004).

7.4. Future directions

Although our investigation was based on five large datasets, it nevertheless seems necessary and worthwhile to further investigate the structure of intelligence using the solutions-hierarchy approach by conducting new research and by applying the approach more broadly in other intelligence variable sets and samples. For instance, future research could further investigate the memory/perception factor. One possible approach for doing so would be to combine perception tasks from the WOTH90 and THUR41 datasets with the memory tasks from BEAU02 and WOTH90 in one data collection. Another important goal would be to investigate older samples. As we noted in the limitations section, most of the datasets in our investigation consisted of relatively young samples and the structure of intelligence may be subject to change across the lifespan.

The development of models of intelligence has often been based on new methods of multivariate data analysis, especially in the domain of factor analysis. Examples include Spearman's two-factor theory and Thurstone's primary mental abilities. The present study is just another example for the close relationship between multivariate data analysis and models of intelligence. It might therefore be expected that further developments in the domain of factor analysis will further our understanding of intelligence. Since the solutions-hierarchy approach does not identify the hierarchical level of the factors with the causal hierarchy of the factors, it might be conceived to specify causal relations in a second step. This would allow for a specification of causal relations across all levels of the hierarchy (upwards, on the same level, and downwards) according to theoretical assumptions and not simply according to the hierarchical level of the factors.

8. Conclusion

In this article, we have suggested that studying hierarchies of factor solutions can provide novel insights into the structure of psychometric intelligence. In our reanalysis of five datasets, we applied the modified solutions-hierarchy approach to five large scale investigations on the structure of intelligence. Our analyses suggest that intelligence has a 1–2–3–5 hierarchy of factor solutions with a g factor at the top of the hierarchy, gc and gf factors at the second level, a speed-reasoning-knowledge taxonomy at the third level, and a four- or five-factor taxonomy with a speed, a reasoning, a fluency, a knowledge and possibly a memory-perception factor at the fourth or fifth level. We believe that these findings provide a building block for future research on intelligence using the solutions-hierarchy approach and have the potential to contribute to a better understanding of the structure of intelligence.

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